A numerical study on the dimension of an extremely inhomogeneous matter distribution

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Abstract

We have developed an algorithm that numericaly computes the dimension of an extremely inhomogeneous matter distribution, given by a discrete hierarchical metric. With our results it is possible to analise how the dimension of the matter density tends to $d = 3$, as we consider larger samples.

In a previous work [1], we have presented a study on the hierarchical metric

$$dS^2 = dt^2 + g_{11}dx^2 + g_{22}dy^2 + g_{33}dz^2 \ , \tag{1}$$

where the metric is defined on all integers, depending on their decomposition in terms of powers of 2 as

$$\begin{aligned} g_{11}(x) &= a(t)^{2k} \,, && \text{with} \quad x = 2^{k+1}n + 2^k - 1 \\ g_{22}(y) &= a(t)^{2\ell} \,, && \text{with} \quad y = 2^{\ell+1}n + 2^\ell - 1 \\ g_{33}(z) &= a(t)^{2m} \,, && \text{with} \quad z = 2^{m+1}n + 2^m - 1 \,. \end{aligned} \tag{2}$$

We obtained the following expression for the matter density,

$$\begin{aligned} T_{00} \equiv \rho &= \frac{1}{8\pi G}\frac{\dot{a}^2}{a^2}\,(k\ell + \ell m + mk) \\ &\equiv \rho_0(t)\,(k\ell + \ell m + mk) \ , \end{aligned} \tag{3}$$

by means of the Einstein equations. Computing the Christoffel symbols and subsequently the curvature tensor for this metric requires some care, since we are not dealing with derivatives of functions, but differences of functions defined on a discrete space.

We speculated whether such a matter distribution could be described by a fractal. As it turned out, from our preliminary analisis (see [1]), the dimension of the matter density tends slowly to $d = 3$.

Considering the relation

$$\lim_{r\to\infty}\frac{N(r)}{r^d} = K \,, \qquad \text{with } N(r) = \sum_{0<x,y,z<r}\frac{\rho}{\rho_0} \,, \tag{4}$$

where K is some constant, and the constant d is the fractal dimension of the matter density [2], it is easy to see that it implies

$$\ln N(r) = \ln K + d \ln r \,, \tag{5}$$

for large r. We have numerically computed $N(r)$ and the plot $\ln N(r) \times \ln r$, in figure (1), showing that equation (5), although valid at very large r), is a good approximation for the behavior of the data.

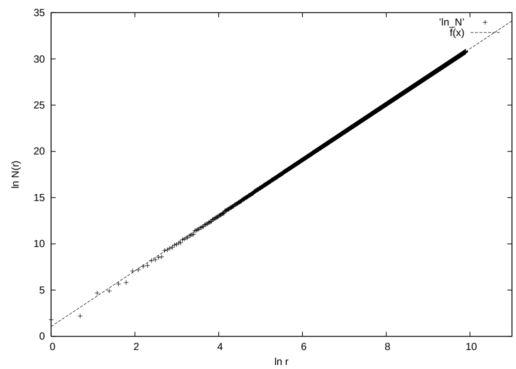

Figure 1: In this graphic we have $\ln N(r) \times \ln r$. The equation of the straight line $f(x) = ax+b$ has $a = 3.00374 \pm 0.000025$, $b = 1.06288 \pm 0.000225$.

Numerical fits for these points, taking increasingly larger samples, have given results for d that approach $d = 3$, as shown in figure (2).

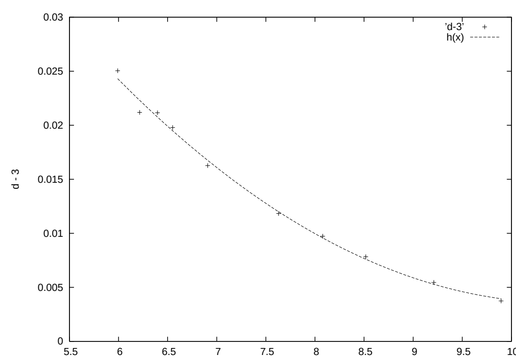

Figure 2: This graphic shows $(d-3) \times \ln r$, obtained for r values up to 20.000. The curve is a parabola fitted to the points by Gnuplot, with equation $h(x) = ax^2 + bx + c$: $a = 0.00102 \pm 0.00014$, $b = -0.02134 \pm 0.00220$, $c = 0.11576 \pm 0.008441$.

As a conclusion, we can say, now, that the dimension of the matter density that generates metric (1) tends to $d = 3$ as shown in figure (2).

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References

[1] E. Abdalla, C. B. M. H. Chirenti, *Phys. A* **337**, 117-122 (2004) .

[2] B. B. Mandelbrot, *The Fractal Geometry of Nature*, New York, 1983.